

# Package ‘GaussianHMM1d’

January 20, 2025

**Title** Inference, Goodness-of-Fit and Forecast for Univariate Gaussian Hidden Markov Models

**Version** 1.1.1

**Description** Inference, goodness-of-fit test, and prediction densities and intervals for univariate Gaussian Hidden Markov Models (HMM). The goodness-of-fit is based on a Cramer-von Mises statistic and uses parametric bootstrap to estimate the p-value. The description of the methodology is taken from Chapter 10.2 of Remillard (2013) <[doi:10.1201/b14285](https://doi.org/10.1201/b14285)>.

**Depends** R (>= 3.5.0), doParallel, parallel, foreach, stats

**License** GPL (>= 2)

**Encoding** UTF-8

**RoxygenNote** 7.2.3

**NeedsCompilation** yes

**Author** Bouchra R. Nasri [aut, cre, cph],  
Bruno N Remillard [aut, ctb, cph]

**Maintainer** Bouchra R. Nasri <[bouchra.nasri@umontreal.ca](mailto:bouchra.nasri@umontreal.ca)>

**Repository** CRAN

**Date/Publication** 2023-07-08 10:50:10 UTC

## Contents

EstHMM1d . . . . .	2
EstRegime . . . . .	3
ForecastHMMeta . . . . .	4
ForecastHMMPdf . . . . .	5
GaussianMixtureCdf . . . . .	6
GaussianMixtureInv . . . . .	6
GaussianMixturePdf . . . . .	7
GofHMM1d . . . . .	8
Sim.HMM.Gaussian.1d . . . . .	9
Sim.Markov.Chain . . . . .	10
SimHMMGaussianInv . . . . .	10
Sn . . . . .	11

EstHMM1d

*Estimation of a univariate Gaussian Hidden Markov Model (HMM)***Description**

This function estimates parameters ( $\mu$ ,  $\sigma$ ,  $Q$ ) of a univariate Hidden Markov Model. It computes also the probability of being in each regime, given the past observations ( $\eta$ ) and the whole series ( $\lambda$ ). The conditional distribution given past observations is applied to obtain pseudo-observations  $W$  that should be uniformly distributed under the null hypothesis. A Cramér-von Mises test statistic is then computed.

**Usage**

```
EstHMM1d(y, reg, max_iter = 10000, eps = 1e-04)
```

**Arguments**

<code>y</code>	( $n \times 1$ ) vector of data
<code>reg</code>	number of regimes
<code>max_iter</code>	maximum number of iterations of the EM algorithm; suggestion 10 000
<code>eps</code>	precision (stopping criteria); suggestion 0.0001.

**Value**

<code>mu</code>	estimated mean for each regime
<code>sigma</code>	estimated standard deviation for each regime
<code>Q</code>	( $reg \times reg$ ) estimated transition matrix
<code>eta</code>	( $n \times reg$ ) probabilities of being in regime $k$ at time $t$ given observations up to time $t$
<code>lambda</code>	( $n \times reg$ ) probabilities of being in regime $k$ at time $t$ given all observations
<code>cvm</code>	Cramér-von Mises statistic for the goodness-of-fit test
<code>U</code>	Pseudo-observations that should be uniformly distributed under the null hypothesis of a Gaussian HMM
<code>LL</code>	Log-likelihood

**Author(s)**

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

**References**

Chapter 10.2 of B. Rémillard (2013). Statistical Methods for Financial Engineering, Chapman and Hall/CRC Financial Mathematics Series, Taylor & Francis.

**Examples**

```
Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2); mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05)
data <- Sim.HMM.Gaussian.1d(mu,sigma,Q,eta0=1,100)$x
est <- EstHMM1d(data, 2, max_iter=10000, eps=0.0001)
```

---

EstRegime

*Estimated Regimes for the univariate Gaussian HMM*


---

**Description**

This function computes and plots the most likely regime for univariate Gaussian HMM using probabilities of being in regime  $k$  at time  $t$  given all observations ( $\lambda$ ) and probabilities of being in regime  $k$  at time  $t$  given observations up to time  $t$  ( $\eta$ ).

**Usage**

```
EstRegime(t, y, lambda, eta)
```

**Arguments**

<code>t</code>	(nx1) vector of dates (years, ...); if no dates then $t=[1:\text{length}(y)]$
<code>y</code>	(nx1) vector of data;
<code>lambda</code>	(nxreg) probabilities of being in regime $k$ at time $t$ given all observations;
<code>eta</code>	(nxreg) probabilities of being in regime $k$ at time $t$ given observations up to time $t$ ;

**Value**

<code>A</code>	Estimated Regime using $\lambda$
<code>B</code>	Estimated Regime using $\eta$
<code>runsA</code>	Estimated number of runs using $\lambda$
<code>runsB</code>	Estimated number of runs using $\eta$
<code>pA</code>	Graph for the estimated regime for each observation using $\lambda$
<code>pB</code>	Graph for the estimated regime for each observation using $\eta$

**Author(s)**

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

**References**

Chapter 10.2 of B. Rémillard (2013). Statistical Methods for Financial Engineering, Chapman and Hall/CRC Financial Mathematics Series, Taylor & Francis.

**Examples**

```

Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2); mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05);
data <- Sim.HMM.Gaussian.1d(mu,sigma,Q,eta0=1,100)$x
t=c(1:100);
est <- EstHMM1d(data, 2)
EstRegime(t,data,est$lambda, est$eta)

```

ForecastHMMeta

*Estimated probabilities of the regimes given new observations***Description**

This function computes the estimated probabilities of the regimes for a Gaussian HMM given new observation after time  $n$ . it also computes the associated weight of the Gaussian mixtures that can be used for forecasted density, cdf, or quantile function.

**Usage**

```
ForecastHMMeta(ynew, mu, sigma, Q, eta)
```

**Arguments**

ynew	new observations (mx1);
mu	vector of means for each regime (r x 1);
sigma	vector of standard deviations for each regime (r x 1);
Q	transition probability matrix (r x r);
eta	vector of the estimated probability of each regime (r x 1) at time n;

**Value**

etanew	values of the estimated probabilities at times $n+1$ to $n+m$ , using the new observations
w	weights of the mixtures for periods $n+1$ to $n+m$

**Author(s)**

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

**References**

Chapter 10.2 of B. Rémillard (2013). Statistical Methods for Financial Engineering, Chapman and Hall/CRC Financial Mathematics Series, Taylor & Francis.

**Examples**

```
mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05); Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2); eta <- c(.1, .9);
x <- c(0.2,-0.1,0.73)
out <- ForecastHMMeta(x,mu,sigma,Q,eta)
```

ForecastHMMPdf

*Density function of a Gaussian HMM at time n+k***Description**

This function computes the density function of a Gaussian HMM at time n+k, given observation up to time n.

**Usage**

```
ForecastHMMPdf(x, mu, sigma, Q, eta, k)
```

**Arguments**

x	points at which the density function is computed (mx1);
mu	vector of means for each regime (r x 1);
sigma	vector of standard deviations for each regime (r x 1);
Q	transition probability matrix (r x r);
eta	vector of the estimated probability of each regime (r x 1) at time n;
k	time of prediction.

**Value**

f	values of the density function at time n+k
w	weights of the mixture

**Author(s)**

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

**References**

Chapter 10.2 of B. Rémillard (2013). Statistical Methods for Financial Engineering, Chapman and Hall/CRC Financial Mathematics Series, Taylor & Francis.

**Examples**

```
mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05); Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2) ;
eta <- c(.9, .1);
x <- seq(-1, 1, by = 0.01)
out <- ForecastHMMPdf(x,mu,sigma,Q,eta,3)
plot(x,out$f,type="l")
```

---

GaussianMixtureCdf      *Distribution function of a mixture of Gaussian univariate distributions*

---

**Description**

This function computes the distribution function of a mixture of Gaussian univariate distributions

**Usage**

```
GaussianMixtureCdf(x, mu, sigma, w)
```

**Arguments**

x	Points at which the distribution function is computed (nx1);
mu	vector of means for each regime (r x 1);
sigma	vector of standard deviations for each regime (r x 1);
w	vector of the probability of each regime (r x r).

**Value**

F	values of the distribution function
---	-------------------------------------

**Author(s)**

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

**Examples**

```
mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05); w <-c(0.8, 0.2);
x <- seq(-1, 1, by = 0.01)
F <- GaussianMixtureCdf(x,mu,sigma,w)
plot(x,F,type="l")
```

---

GaussianMixtureInv      *Inverse distribution function of a mixture of Gaussian univariate distributions*

---

**Description**

This function computes the inverse distribution function of a mixture of Gaussian univariate distributions

**Usage**

```
GaussianMixtureInv(p, mu, sigma, w)
```

**Arguments**

p	Points in (0,1) at which the distribution function is computed (n x 1);
mu	vector of means for each regime (r x 1);
sigma	vector of standard deviations for each regime (r x 1);
w	vector of the probability of each regime (r x 1).

**Value**

q	values of the quantile function
---	---------------------------------

**Author(s)**

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

**Examples**

```
mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05); w <-c(0.8, 0.2);  
p <- seq(0.01, 0.99, by = 0.01)  
q <- GaussianMixtureInv(p,mu,sigma,w)  
plot(p,q,type="l")
```

---

GaussianMixturePdf      *Density function of a mixture of Gaussian univariate distributions*

---

**Description**

This function computes the density function of a mixture of Gaussian univariate distributions

**Usage**

```
GaussianMixturePdf(x, mu, sigma, w)
```

**Arguments**

x	Points at which the density is computed (n x 1);
mu	vector of means for each regime (r x 1);
sigma	vector of standard deviations for each regime (r x 1);
w	vector of the probability of each regime (r x 1).

**Value**

f	Values of the distribution function
---	-------------------------------------

**Author(s)**

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

**Examples**

```
mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05); w <-c(0.8, 0.2);
x <- seq(-1, 1, by = 0.01)
f <- GaussianMixturePdf(x,mu,sigma,w)
plot(x,f,type="l")
```

GofHMM1d

*Goodness-of-fit test of a univariate Gaussian Hidden Markov Model***Description**

This function performs a goodness-of-fit test of a Gaussian HMM based on a Cramér-von Mises statistic using parametric bootstrap.

**Usage**

```
GofHMM1d(y, reg, max_iter = 10000, eps = 1e-04, n_sample = 1000, n_cores)
```

**Arguments**

y	(n x 1) data vector
reg	number of regimes
max_iter	maximum number of iterations of the EM algorithm; suggestion 10 000
eps	eps (stopping criteria); suggestion 0.0001
n_sample	number of bootstrap samples; suggestion 1000
n_cores	number of cores to use in the parallel computing

**Value**

pvalue	pvalue of the Cramér-von Mises statistic in percent
mu	estimated mean for each regime
sigma	estimated standard deviation for each regime
Q	(reg x reg) estimated transition matrix
eta	(n x reg) conditional probabilities of being in regime k at time t given observations up to time t
lambda	(n x reg) probabilities of being in regime k at time t given all observations
cvm	Cramér-von Mises statistic for the goodness-of-fit test
W	Pseudo-observations that should be uniformly distributed under the null hypothesis of a Gaussian HMM
LL	Log-likelihood



**Author(s)**

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

**References**

Chapter 10.2 of B. Rémillard (2013). Statistical Methods for Financial Engineering, Chapman and Hall/CRC Financial Mathematics Series, Taylor & Francis.

**Examples**

```
Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2); mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05)
data <- Sim.HMM.Gaussian.1d(mu,sigma,Q,eta0=1,100)$x
gof <- GofHMM1d(data, 2, max_iter=10000, eps=0.0001, n_sample=100,n_cores=2)
```

---

Sim.HMM.Gaussian.1d     *Simulation of a univariate Gaussian Hidden Markov Model (HMM)*

---

**Description**

This function simulates observations from a univariate Gaussian HMM

**Usage**

```
Sim.HMM.Gaussian.1d(mu, sigma, Q, eta0, n)
```

**Arguments**

mu	vector of means for each regime (r x 1);
sigma	vector of standard deviations for each regime (r x 1);
Q	Transition probability matrix (r x r);
eta0	Initial value for the regime;
n	number of simulated observations.

**Value**

x	Simulated Data
reg	Markov chain regimes

**Author(s)**

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

**Examples**

```
Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2) ; mu <- c(-0.3 ,0.7) ; sigma <- c(0.15,0.05);
sim <- Sim.HMM.Gaussian.1d(mu,sigma,Q,eta0=1,n=100)
```

---

Sim.Markov.Chain      *Simulation of a finite Markov chain*

---

**Description**

This function generates a Markov chain  $X(1), \dots, X(n)$  with transition matrix  $Q$ , starting from a state  $\text{eta0}$ .

**Usage**

```
Sim.Markov.Chain(Q, n, eta0)
```

**Arguments**

$Q$	Transition probability matrix ( $r \times r$ );
$n$	length of series;
$\text{eta0}$	initial value in $1, \dots, r$ .

**Value**

$x$	Simulated Markov chain
-----	------------------------

**Author(s)**

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

**Examples**

```
Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2) ;
sim <- Sim.Markov.Chain(Q,eta0=1,n=100)
```

---

SimHMMGaussianInv      *Simulation of a univariate Gaussian Hidden Markov Model (HMM)*

---

**Description**

Generates a univariate regime-switching random walk with Gaussian regimes starting from a given state  $\text{eta0}$ , using the inverse method from noise  $u$ . Can be useful when generating multiple time series.

**Usage**

```
SimHMMGaussianInv(u, mu, sigma, Q, eta0)
```

**Arguments**

u	series of uniform i.i.d. series (n x 1);
mu	vector of means for each regime (r x 1);
sigma	vector of standard deviations for each regime (r x 1);
Q	Transition probability matrix (r x r);
eta0	Initial value for the regime;

**Value**

x	Simulated Data
eta	Probability of regimes

**Author(s)**

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

**References**

Nasri & Remillard (2019). Copula-based dynamic models for multivariate time series. *JMVA*, vol. 172, 107–121.

**Examples**

```
Q <- matrix(c(0.8, 0.3, 0.2, 0.7),2,2)
set.seed(1)
u <-runif(250)
mu <- c(-0.3 ,0.7)
sigma <- c(0.15,0.05);
eta0=1
x <- SimHMMGaussianInv(u,mu,sigma,Q,eta0)
```

---

Sn	<i>Cramer-von Mises statistic for goodness-of-fit of the null hypothesis of a univariate uniform distribution over [0,1]</i>
----	--

---

**Description**

This function computes the Cramér-von Mises statistic Sn for goodness-of-fit of the null hypothesis of a univariate uniform distribution over [0,1]

**Usage**

Sn(U)

**Arguments**

U                      vector of pseudos-observations (approximating uniform variates)

**Value**

$S_n$                       Cramér-von Mises statistic

**Author(s)**

Bouchra R Nasri and Bruno N Rémillard, January 31, 2019

# Index

EstHMM1d, [2](#)  
EstRegime, [3](#)

ForecastHMMeta, [4](#)  
ForecastHMMPdf, [5](#)

GaussianMixtureCdf, [6](#)  
GaussianMixtureInv, [6](#)  
GaussianMixturePdf, [7](#)  
GofHMM1d, [8](#)

Sim.HMM.Gaussian.1d, [9](#)  
Sim.Markov.Chain, [10](#)  
SimHMMGaussianInv, [10](#)  
Sn, [11](#)